Interference Alignment in Dual-Hop MIMO Interference Channel

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Abstract—With interference alignment in the spatial domain, the achievable degrees of freedom (DoF) of a single-hop multiple-input multiple-output (MIMO) interference channel (IC) are limited by the number of antennas at the sources and destinations. The use of relays introduces additional freedoms to manage the interference and can enhance the DoF performance. However, the characterization of the DoF regions with relays is much more complicated and is not available in the literature. In this paper, we shall investigate the DoF of the dual-hop MIMO IC via interference alignment. Based on the solvability of the alignment conditions, the upper bound for the maximum achievable DoF tuple is obtained. To evaluate the tightness of the derived bound, we further propose an iterative algorithm to determine the processing matrices at the sources, relays, and destinations for a given feasible DoF tuple. It is shown that the proposed algorithm can achieve the upper bound for the sum DoF in the low and high DoF regions, where the achievability indicates that the upper bound indeed gives the maximum sum DoF. It is also found that despite the DoF loss caused by the half-duplexity assumption, the dual-hop IC with sufficient number of relays can still outperform the conventional single-hop IC under most circumstances.

Index Terms—Dual-hop MIMO interference channel, interference alignment, degrees of freedom.

I. INTRODUCTION

The characterization of the capacity region for interference channel (IC) has been an open problem for decades. In the high signal-to-noise ratio (SNR) region, the capacity can be characterized by the degrees of freedom (DoF), also known as the capacity pre-log factor or the multiplexing gain. Recent results show that interference alignment is a promising technique to achieve the optimal DoF for some ICs [1], [2]. In a $K$-user time-varying or frequency-selective IC where all nodes are equipped with $M$ antennas, $KM/2$ DoF can be achieved almost surely by interference alignment with symbol extensions [3]. This indicates that, in the high SNR region, the capacity can grow linearly with the number of users and the number of antennas. However, symbol extension is not always applicable in practice and will increase the implementation complexity of the transceivers. Accordingly, MIMO interference alignment [4] was proposed where the alignment is only implemented in the spatial domain with multi-antenna transceivers. For this case, the achievable sum DoF becomes saturated and cannot increase linearly with $K$ [5]–[7]. This is because, with interference alignment in the spatial domain, the available signal dimension is limited by the number of antennas, which is quite small compared to that of the time-varying or frequency-selective channels. Such limited channel freedoms will cause a dramatic decrease in the sum DoF.

One potential solution to address this issue is to use relays. With the additional freedoms provided by relays, we can optimize the equivalent channels from the sources to the destinations by proper signal design. By doing so, part of the interference between different links can be aligned and canceled by the relays. As a result, the dimension of the remaining interference at the destinations is reduced. However, the achievable DoF region of multi-hop ICs is more difficult to determine. In the literature, this problem has been investigated with and without the direct links. With direct links, [8], [9] showed that adding amplify-and-forward (AF) relays can reduce the required number of symbol extensions, but does not result in any DoF gains [10]. Without direct links, a $2 \times 2 \times 2$ network (2 sources, 2 relays and 2 destinations) with single-antenna nodes is considered in [11], and it was shown that the min-cut outer bound of 2 DoF (excluding the duplex factor) can be achieved with symbol extensions. [12] extended the result in [11] to the case with multi-antenna nodes. With more than two pairs of users, [13] investigated the DoF region of a class of multi-hop time-varying IC. In [14], it was shown that with real interference alignment [15], a fully connected $K \times K \times K$ network can almost surely achieve the DoF of $K$. However, in practice, the estimated channel coefficients cannot satisfy the requirement of the infinite precision for the real interference alignment and thus the DoF of $K$ is not achievable. Besides directly characterizing the DoF regions, some work tried to address the problem from a practical perspective by designing the beamforming matrices at the relays and/or sources to maximize the sum-rate [16]–[18] or minimize the transmit power with Quality of Service (QoS) requirements [19]–[21]. However, due to the nonconvex nature of the problem, the optimality of the above algorithms is not guaranteed and the achieved DoF is not clear.

It can be observed that without the direct links, the achievable DoF is still unknown for a general dual-hop MIMO IC. In this paper, we will consider a fully connected $K_1 \times K_2 \times K_1$ network with half-duplex AF relays as shown in Fig. 1. Our objective is to determine the achievable DoF region of the concerned network with interference alignment in the spatial
domain. We shall first determine the interference alignment feasibility conditions to achieve a given DoF tuple. By studying the solvability of the interference alignment conditions, an upper bound for the maximum achievable DoF tuple is derived with general antenna settings. For the case where the sources and/or destinations have the same number of antennas, closed-form upper bounds for the sum DoF are provided. Based on the upper bound of the achievable DoF tuple, the minimum number of relaying antennas grows quadratically with the number of S-D pairs and the number of user antennas, but grows linearly with them when multi-antenna relays are used.

To evaluate the tightness of the derived bounds, we further propose an iterative algorithm for a given DoF tuple to determine the processing matrices at the sources, relays and destinations. The proposed algorithm minimizes the interference leakage, and can create interference-free channels for each S-D pair. Simulation results will show that the proposed algorithm achieves the upper bound of the sum DoF in the low and high DoF regions. This indicates that the derived upper bound provides the exact maximum achievable sum DoF and the proposed algorithm can achieve this maximum under such circumstances. It is also observed that, although half of the DoF is lost due to the half-duplexity assumption, the dual-hop IC still can outperform the single-hop IC under many circumstances. The DoF improvement is more significant when the system has a symmetric antenna setting or when the number of S-D pairs is much larger than the number of user antennas.

The rest of this paper is organized as follows. In Section II, the system model is introduced. The upper bound of the achievable DoF tuple is derived in Section III, and an iterative alignment algorithm is presented in Section IV to check the tightness of the bound. Numerical results are shown in Section V, and the paper concludes with Section VI.

II. SYSTEM MODEL

Consider a $K_1 \times K_2 \times K_3$ dual-hop IC as shown in Fig. 1, where $K_1$ sources ($S_m, m = 1, 2, \cdots, K_1$) transmit information to their dedicated destinations ($D_n, n = 1, 2, \cdots, K_3$), through $K_2$ AF relay nodes. The direct links between the sources and destinations are ignored due to the long transmission distance. Half-duplex relays are employed, so that the two hop transmissions are operating in two different time slots or frequency bands. We assume that the source $S_m$ and destination $D_n$ are equipped with $M_m$ and $N_n$ antennas, respectively, while the $K_2$ relays each have one antenna. The case with multi-antenna relays will be discussed in a later section. Assume that all the sources, relays, and destinations have the channel knowledge of the whole system [1], [7], [13]. The channel coefficient vectors from $S_m$ to $R_l$ and from $R_l$ to $D_n$ are denoted by $f_{lm}$ and $g_{nl}$, respectively. All channel coefficients of $f_{lm}$ and $g_{nl}$ are assumed to be generic (e.g., drawn from a continuous probability distribution).

For the $m$th S-D pair, $d_m$ independent data streams are transmitted and denoted by $s_m \in \mathbb{C}^{d_m \times 1}$. The DoF tuple of the system is defined by $(\frac{d_{11}}{2}, \frac{d_{12}}{2}, \cdots, \frac{d_{K_1}}{2})$, where $\frac{1}{2}$ comes from the half-duplexity assumption. In the first hop, all sources transmit simultaneously, and the received signal at the relay $R_l$, $l = 1, \cdots, K_2$, is given by

$$y_{l,1} = \sum_{m=1}^{K_1} f_{lm}^T V_m s_m + n_{l,1},$$

where $V_m \in \mathbb{C}^{M_m \times d_m}$ denotes the linear precoding matrix at $S_m$, and $n_{l,1} \sim \mathcal{CN}(0, I)$ represents the additive white Gaussian noise (AWGN) with zero mean and unit variance.

In the second hop, each relay $R_l$ amplifies its received signal with a factor $w_l$ and retransmits it to the destinations simultaneously. Thus, the received signal at the destination $D_n$, $n = 1, \cdots, K_3$, is given by

$$y_{n,2} = \sum_{l=1}^{K_2} g_{nl} w_l y_{l,1} + n_{n,2},$$

where $n_{n,2} \sim \mathcal{CN}(0, I)$ represents the circularly symmetric AWGN vector with zero mean and unit variance. By substituting (1) to (2), we obtain the end-to-end signal model as

$$y_{n,2} = \sum_{l=1}^{K_2} g_{nl} w_l \sum_{m=1}^{K_1} f_{lm}^T V_m s_m + \sum_{m=1, m \neq n}^{K_2} \sum_{l=1}^{K_2} g_{nl} w_l f_{lm}^T V_m s_m$$

$$+ \sum_{l=1}^{K_2} g_{nl} w_l n_{l,1} + n_{n,2}$$

$$= H_{nm} V_n s_n + \sum_{m=1, m \neq n}^{K_1} H_{nm} V_m s_m$$

$$+ \sum_{l=1}^{K_2} g_{nl} w_l n_{l,1} + n_{n,2},$$

where the equivalent channel matrix from $S_m$ to $D_n$ is defined as $H_{nm} \triangleq \sum_{l=1}^{K_2} g_{nl} w_l f_{lm}^T$, $m$ and $n = 1, \cdots, K_1$. Denote the processing matrix at $D_n$ by $U_n \in \mathbb{C}^{K_3 \times d_n}$.

The interference alignment feasibility conditions for a DoF tuple $(\frac{d_{11}}{2}, \frac{d_{12}}{2}, \cdots, \frac{d_{K_1}}{2})$ are then given by

$$U_n^T H_{nm} V_m = 0, \forall m \neq n$$

and

$$\text{rank} \left( U_n^T H_{nn} V_n \right) = d_n, \forall n.$$
Conditions in (4) ensure that all the interfering signals at $D_n$ are aligned in a subspace of $N_n - d_m$ dimensions, and can be forced to zero by $U_n$. Conditions in (5) guarantee that, after zero-forcing, the remaining signal space at $D_n$ has a dimension of $d_m$. When both conditions in (4) and (5) are satisfied, the given DoF tuple $\left( \frac{d_1}{2}, \frac{d_2}{2}, \cdots, \frac{d_K}{2} \right)$ is achieved.

III. UPPER BOUNDING THE ACHIEVABLE DOF TUPLE

Determining the feasibility of a given DoF tuple $\left( \frac{d_1}{2}, \frac{d_2}{2}, \cdots, \frac{d_K}{2} \right)$ is equivalent to studying the solvability of the conditions in (4) and (5). We shall therefore derive the upper bound for the achievable DoF tuple $\left( \frac{d_1}{2}, \frac{d_2}{2}, \cdots, \frac{d_K}{2} \right)$ by investigating the solvability of these equations. For single-hop MIMO IC with generic channel matrices, $V_n$ and $U_n$ determined from (4) do not depend on the intended channel coefficient matrices $H_{mn}$, and thus, the conditions in (5) hold with probability one. However, it is not true for the dual-hop MIMO IC because that the coefficients in (4) and (5) are dependent on each other. On the other hand, relaxing the feasibility conditions in (5) will not affect the validity of the following proposed upper bounds, since they give the necessary conditions for a DoF tuple being achievable. Thus, we will only consider the feasibility conditions in (4) in the following discussion.

We begin by showing that for the case with $\max \{ M_m, N_m \} \geq K_2$, further increasing $M_m$ and $N_m$ cannot improve the achievable DoF tuple. We shall first prove so when the destination has more antennas than $K_2$. Since the sum DoF of all users should not exceed the rank of the relaying channels, i.e., $\sum_{m=1}^{K_2} d_m \leq K_2$, the spatial dimensions occupied by the desired signals and the interfering signals will not be larger than $K_2$ as well. The extra antennas, that exceed the number of $K_2$, can only provide some diversity gains but no DoF. The same is true when the source has more antennas than $K_2$. Therefore, in the following discussions, we assume that all the source and destination nodes are equipped with $K_2$ antennas at most, i.e., $\max \{ M_m, N_m \} \leq K_2$ for all $m$.

A. Upper Bound of the Achievable DoF Tuple

To determine an upper bound of the achievable DoF tuple by interference alignment, a group of necessary conditions are given in the following theorem.

**Theorem 1:** Consider a $K_1 \times K_2 \times K_1$ dual-hop MIMO IC. The necessary conditions for the achievable DoF tuple $\left( \frac{d_1}{2}, \frac{d_2}{2}, \cdots, \frac{d_K}{2} \right)$ by interference alignment are given by

$$
\min \{ M_m, N_m \} \geq d_m, \forall m,
$$

$$
K_2 \geq \sum_{m=1}^{K_2} d_m,
$$

$$
\sum_{m \in K} d_m (M_m + N_m - 2d_m) + (K_2 - 1) \geq \sum_{m,n \in K, m \neq n} d_md_n
$$

for all subset $K \subseteq \{1, 2, \cdots, K_1\}$.

The first two constraints in Theorem 1 are obtained by information theoretic outer bounds, where (6) can be derived in a straightforward way from (5), and (7) is due to the fact that the sum DoF of all users should not exceed the freedom of the relaying channels. The third constraint in (8) implies the properness of the polynomial system in (4). Specifically, a polynomial system is called proper if the number of equations is no larger than the number of variables. Otherwise, the system is improper [5]. The left and right hand sides of (8) represent the number of valid variables and number of equations of the polynomial system in (4), respectively. The first term of the left-hand side (LHS) denotes the total number of valid variables contributed by the processing matrices at the sources and destinations, i.e., $V_m$ and $U_m$. The second term denotes the number of valid variables contributed by the relays, i.e., $w_l$.

The solvability and the properness of a polynomial system are not equivalent in general. In the following lemma, we prove that properness, i.e., the constraint in (8), is indeed the necessary condition for the solvability of the polynomial system in (4).

**Lemma 1:** Improperness of the polynomial system in (4) implies its unsolvability with probability 1.

The proof is provided in Appendix A.

Besides providing the upper bound of the DoF tuple of the general system, Theorem 1 can also be utilized to bound the sum DoF for the simplified systems with equal antenna settings at the sources and/or destinations, as shown in the following two corollaries.

**Corollary 1:** For an asymmetric case where all source nodes are equipped with $M$ antennas and all destination nodes are equipped with one antenna, an upper bound for the sum DoF is given by

$$
\text{SDoF} \leq \frac{1}{2} \min \left\{ K_1, K_2, \sqrt{K_2 - 1 + \frac{M^2}{4} + \frac{M}{2}} \right\}.
$$

**Corollary 2:** For a symmetric case where all source and destination nodes are equipped with $M$ antennas, an upper bound for the sum DoF is given by

$$
\text{SDoF} \leq \frac{1}{2} \min \left\{ K_1 M, K_2, \sqrt{K_2 - 1 + \left( M - \frac{1}{2} \right)^2 + M - \frac{1}{2}} \right\}.
$$

Corollary 1 is a straightforward result from Theorem 1, and also holds when all destination nodes are equipped with multiple antennas. The bound in Corollary 2 is derived with a relaxation of $\sum_{m \in K} d_m \leq \sum_{m \in K} d_m^2$.

B. Minimum Required Number of Relays for a Given DoF Tuple

By combining the conditions (7) and (8) in Theorem 1, we can obtain the minimum required number of relays for a given DoF tuple.
Corollary 3: Consider a $K_1 \times K_2 \times K_1$ dual-hop MIMO IC. To achieve a DoF tuple \( \left( \frac{d_1}{2}, \frac{d_2}{2}, \ldots, \frac{d_{K_1}}{2} \right) \) by interference alignment, the minimum required number of single-antenna relays is obtained as

\[
K_2 \geq \max \left\{ \left( \frac{K_1}{m=1} d_m \right)^2 - \sum_{m=1}^{K_1} d_m (M_m + N_m - d_m) + 1, \sum_{m=1}^{K_1} d_m \right\},
\]

Note that when $d_m = M_m = N_m = M$ for all $m = 1, 2, \ldots, K_1$, each S-D pair achieves the full DoF, and the required number of relays is given by

\[
K_2 \geq M^2 K_1 (K_1 - 1) + 1.
\]

Note that this agrees with the $M = 1$ case in [22].

C. Multi-antenna Relays

In this subsection, we consider the case where the relays are equipped with multiple antennas. Given their ability to jointly process the signals received from different antennas, we expect that the use of multi-antenna relays can reduce the minimum required number of relaying antennas for a given DoF tuple. By following the same idea as that utilized to derive Theorem 1, i.e., to determine the solvability of the polynomial equations in (4), we can obtain the following proposition.

Proposition 1: Consider a $K_1 \times K_2 \times K_1$ dual-hop MIMO IC where all channel matrices are assumed to be generic and no symbol extension is applied. Source $S_m$, relay $R_l$, and destination $D_n$ are equipped with $M_m$, $L_l$, $N_n$ antennas, respectively, where $m, n = 1, 2, \ldots, K_1$ and $l = 1, 2, \ldots, K_2$. Then, any DoF tuple $\left( \frac{d_1}{2}, \frac{d_2}{2}, \ldots, \frac{d_{K_1}}{2} \right)$ achievable with interference alignment needs to satisfy the following constraints

\[
\min \{ M_m, N_m \} \geq d_m, \forall m, \tag{13}
\]

\[
\sum_{l=1}^{K_2} L_l \geq \sum_{m=1}^{K_1} d_m, \tag{14}
\]

\[
\sum_{m \in K} d_m (M_m + N_m - 2d_m) + \left( \sum_{l=1}^{K_2} L_l^2 - 1 \right) \geq \sum_{m, n \in K, m \neq n} d_m d_n, \tag{15}
\]

for all subset $K \subseteq \{1, 2, \ldots, K_1\}$.

Similar to Theorem 1, the third condition (15) in Proposition 1 relies on Lemma 1. We cannot prove the lemma for this general case. However, we did not witness any counter examples during our extensive simulations so far. For the case when $d_m = M_m = N_m = M$ and $K_2 = 1$, the minimum required number of relaying antennas is $L_1 = MK_1$, which grows linearly with $M$ and $K_1$. By comparing this result with that given in (12), we can observe that the employment of the multi-antenna relays can indeed save a substantial amount of relaying antennas.

IV. Iterative Algorithm for Interference Alignment

In this section, an iterative algorithm will be proposed to determine the processing matrices at all nodes to achieve a given DoF tuple $\left( \frac{d_1}{2}, \frac{d_2}{2}, \ldots, \frac{d_{K_1}}{2} \right)$, which provides a way to check the tightness of the derived bound. The idea is to minimize the total interference power that leaks into the users’ desired signal subspace [8]. If the total interference leakage converges to zero, the desired signal subspace will be interference-free and the given DoF tuple is feasible.

A. Interference Leakage

It follows from (3) that the total interference leakage at $D_n$ due to all undesired sources is given by

\[
I_n = \text{Tr} \left\{ U_n^T Q_n U_n^H \right\},
\]

where

\[
Q_n = \sum_{m=1, m \neq n}^{K_1} \frac{1}{d_m} H_{nm} V_m V_m^H H_{nm}^H
\]

is the interference covariance matrix at $D_n$. Define $w = [w_1, w_2, \ldots, w_{K_2}]^T$. If we can find a group of $\{U_n\}$, $\{V_m\}$ and $w$ to make the sum interference leakage at all destinations zero, then the conditions in (4) are satisfied. Given that a closed form solution is extremely difficult to determine, we propose an iterative algorithm where the optimization problem can be formulated as follows

\[
\min_{\{U_n\}, \{V_m\}, w} \sum_{n=1}^{K_1} I_n
\]

s.t.

\[
V_m^H V_m = I_{d_m}, \forall m,
\]

\[
U_n^H U_n = I_{d_n}, \forall n,
\]

\[
w^H w = 1.
\]

Note that the objective function in (18) is always larger than or equal to zero. We adopt the sum power constraint at all relays in (18). With the sum power constraint, the achievable rate of the network should be better than or equal to that with individual power constraints given the same total power. Since we want to evaluate the tightness of the proposed upper bounds, the sum power constraint will be preferred. The constraints given in (18) will naturally lead to the proposed iterative algorithm.

B. Iterative Algorithm

An alternating minimization procedure has been proposed in [23]. The basic idea is to minimize the objective function with respect to one variable at a time while keeping the others fixed. By doing so, we can obtain $\{U_n\}$, $\{V_m\}$ and $w$ according to the following three steps:
\[ A_{nm} = \begin{bmatrix} f_{1m} V_{n}^{*} V_{m}^{T} f_{1n} g_{n1}^{H} U_{n}^{*} U_{n}^{T} g_{n1} & \cdots & f_{1m} V_{n}^{*} V_{m}^{T} f_{K_{2}m} g_{nK_{2}}^{H} U_{n}^{*} U_{n}^{T} g_{nK_{2}} \\ \vdots & \ddots & \vdots \\ f_{K_{2}m} V_{m}^{*} V_{m}^{T} f_{1n} g_{n1}^{H} U_{n}^{*} U_{n}^{T} g_{n1} & \cdots & f_{K_{2}m} V_{m}^{*} V_{m}^{T} f_{K_{2}m} g_{nK_{2}}^{H} U_{n}^{*} U_{n}^{T} g_{nK_{2}} \end{bmatrix} \]  

(25)

Therefore, \( A_{nm} \) is positive semi-definite. By fixing \( \{ U_{n} \} \) and \( \{ V_{m} \} \), the optimization problem in (18) can be written as

\[
\begin{align*}
\min_{w} & \quad w^{H} \left( \sum_{n=1}^{K_{1}} \sum_{m=1, m \neq n}^{K_{2}} \frac{A_{nm}}{d_{nm}} \right) w \\
\text{s.t.} & \quad w^{H} w = 1.
\end{align*}
\]

(27)

The optimal \( w \) is the least dominant eigenvector of

\[
\sum_{n=1}^{K_{1}} \sum_{m=1, m \neq n}^{K_{2}} \frac{A_{nm}}{d_{nm}}.
\]

The iterative procedure is summarized in Algorithm 1 as follows.

Algorithm 1: Iterative Algorithm for Interference Alignment

Randomly generate \( \{ V_{m} \} \) and \( \{ U_{n} \} \), such that

\[
V_{m}^{H} U_{m} = I_{d_{m}} \text{ and } U_{n}^{H} U_{n} = I_{d_{n}}, \forall m, n.
\]

Begin iteration:

1. Calculate the optimal \( w \) and update all processing matrices. If \( \sum_{n=1}^{K_{1}} I_{n} = 0 \), stop.
2. Calculate the optimal \( \{ V_{m} \} \) and update all processing matrices. If \( \sum_{n=1}^{K_{1}} I_{n} = 0 \), stop.
3. Calculate the optimal \( \{ U_{n} \} \) and update all processing matrices. If \( \sum_{n=1}^{K_{1}} I_{n} = 0 \), stop.
4. Continue until convergence.

C. Convergence and Optimality

Because each step in Algorithm 1 will reduce the sum interference leakage, which is always larger than or equal to zero, the convergence is guaranteed. However, due to the limitation of the alternating minimization procedure and the non-convexity of the problem, the convergence to the global minimum cannot be guaranteed.

There can be multiple local minimums since the objective function in (18) is non-convex, and Algorithm 1 may converge to the global minimum or any one of the local minimums, depending on the initial iteration state. Therefore, whether the algorithm can converge to zero, depends on the feasibility of the given DoF tuple and the initial iteration state. If the objective function in (18) indeed converges to zero, then the global minimum is achieved since the objective function is always non-negative.

V. NUMERICAL RESULTS

In this section, the DoF region achieved by the proposed algorithm in Section IV is compared with the upper bound given by Theorem 1. For ease of illustration, the sum DoF,
instead of the DoF tuple, is utilized as the comparison metric. It should be noted that without symbol extensions, the achieved DoF of each S-D pair is an integer (excluding the duplex factor), including zero. An S-D pair with zero DoF does not transmit, and thus will not cause any interference to other S-D pairs.

Limited by the precision of Matlab, the sum interference leakage cannot be exactly equal to zero. Therefore, we will utilize the fraction of the total interference leakage in the desired signal space as the metric to decide when to stop the iteration in the simulations. Such parameter is defined as

$$p = \frac{\sum_{n=1}^{K_1} I_n}{\sum_{n=1}^{K_1} \text{Tr} \{Q_n\}}.$$  \hspace{1cm} (28)

In our simulation, if \(p\) converges to a value smaller than \(10^{-10}\) and condition (5) is also satisfied, we then state that the given DoF tuple is attained and the interference alignment is feasible.

A. Tightness of the Bounds

The DoF performance for the systems with the symmetric (\(M = N\)) and asymmetric (\(M \neq N\)) antenna settings are shown in Figs. 2 and 3, respectively. The figures illustrate the achievable sum DoF as a function of \(K_2\). For both the symmetric and asymmetric cases, it is observed that the achieved sum DoF by the iterative algorithm is the same as the upper bound in the low and high DoF regions. In the low DoF region, the source and destination nodes have enough extra antennas that can be utilized for interference alignment and cancellation, while the relay nodes can just amplify and forward. In the high DoF region, more relays are needed. Most of the interference can then be neutralized through the multiple relaying links, and the dimension of the remaining interference seen by each destination will be greatly reduced.

B. Comparison with Single-hop Interference Channel

For the conventional single-hop MIMO IC, the sum DoF is upper bounded by \(\left\lfloor \frac{(M+N)K_1}{K_1+1} \right\rfloor\). We compare this upper bound with the sum DoF achieved by the proposed algorithm to show the impact of the relays. The results for the systems with symmetric and asymmetric antenna settings are illustrated in Figs. 4 and 5, respectively. For the symmetric case, it is observed that a higher DoF can be achieved by the dual-hop IC when \(K_2\) is larger than a specific value. For the asymmetric case, the dual-hop IC can still outperform the single-hop IC when \(M = 2\), despite the half-duplexity assumption.
In fact, the condition for the dual-hop IC to outperform the single-hop IC is given by
\[
K_1 \min \left( \frac{M + N}{K_1 + 1} \right) \geq \frac{N}{2}
\]
For the symmetric case with \( M = N \), (29) can be rewritten as
\[
\frac{K_1 M}{2} \geq \frac{K_1 M}{(K_1 + 1)/2},
\]
where the inequality always holds when \( K_1 \geq 3 \), and \( \frac{K_1 M}{2} = \left\lfloor \frac{M + N}{K_1 + 1} \right\rfloor \) only when \( K_1 = 3 \) and \( M \) is even. When \( K_1 = 2 \), the achievable DoF of the single-hop IC is \( M \) [25], which is the same as the maximum DoF of the dual-hop IC. Therefore, for the case with \( M = N \) and sufficient number of relays, the dual-hop IC can achieve a better DoF than the single-hop IC when \( K_1 \geq 4 \) or when \( K_1 = 3 \) and \( M \) is an odd number; otherwise, the two ICs can achieve the same DoF.

For the asymmetric case with \( M \neq N \), by substituting the condition \( \left\lfloor \frac{M + N}{K_1 + 1} \right\rfloor < M + N \) into (29), we can obtain a sufficient condition for the dual-hop IC having a performance gain with large \( K_1 \) as
\[
K_1 \geq \frac{2(M + N)}{\min(M, N)} = 2 \left( 1 + \frac{\max(M, N)}{\min(M, N)} \right),
\]
which indicates that the dual-hop IC more likely achieves a higher DoF when \( K_1 \) is large and the ratio of \( \max(M, N) \) is small. Therefore, the assumption of \( M = N \) in Fig. 4 favors the dual-hop IC the most, while the assumption of \( N = 1 \) in Fig. 5 favors the single-hop IC the most.

**C. Required Number of Relaying Antennas**

For a given sum DoF, we determine the required number of relaying antennas for the system with multiple single-antenna relays and that with a multi-antenna relay in Fig. 6. It can be observed that the theoretical lower bound derived in Corollary 3 is fairly close to the required number of the relaying antennas determined by the iterative algorithm. We can also observe that when the sum DoF is less than or equal to 2, the requirement on the number of relaying antennas of the two systems are the same. Here, the threshold value of 2 corresponds to half of the maximum DoF of the single-hop MIMO IC, i.e., \( \left\lfloor \frac{M + N}{K_1 + 1} \right\rfloor \). This is because, when the sum DoF is less than or equal to 2, the sources and destinations have enough freedoms to align and cancel all the interference, and the relays do not need to mitigate the interference. When the sum DoF is larger than 2, there exists a big performance gap between these two systems. This is because the multi-antenna relay can jointly process the received signals, while the single-antenna relays only share the channel state information.

**VI. CONCLUSIONS**

Relaying is a promising solution to increase the channel freedoms in ICs for the purpose of interference management. In this paper, we investigated the achievable DoF region of the dual-hop MIMO IC. We derived the upper bound for the maximum achievable DoF tuple, and proposed an iterative algorithm to determine the processing matrices at all nodes to attain a given feasible DoF tuple. Simulation results showed that the proposed algorithm can achieve the upper bound of the sum DoF in the low and high DoF regions, which indicates that the derived upper bound is indeed the maximum achievable sum DoF under such circumstances. It was also shown that despite the DoF loss caused by the half-duplexity assumption, the dual-hop MIMO IC can still have a better DoF performance than the conventional single-hop MIMO IC under many circumstances. The upper bound of the achievable DoF tuple was also used to determine the minimum required number of relaying antennas for a certain DoF tuple. It was then shown that with single antenna relays, the required number of relaying antennas grows quadratically with the number of S-D pairs and the number of antennas at the sources and destinations. With multi-antenna relays, the required number of relaying antennas is greatly reduced and grows linearly.

**APPENDIX A**

**Proof:** For simplicity, we only prove the case where \( K = \{1, 2, \cdots, K_1\} \). The proof for any \( K \subset \{1, 2, \cdots, K_1\} \) can be given by a similar procedure. The number of equations in (4) is denoted by \( N_{eq} = \sum_{n=1}^{K_1} \sum_{m=1, m \neq n} d_m d_n \). However, some of the variables in (4) are redundant. We will first show the number of the valid variables is given by \( N_{va} = \sum_{m=1}^{K_1} d_m (M_m + N_m - 2d_m) + (K_2 - 1) \).

Define
\[
\bar{U}_n = G_n^T U_n,
\]
for \( n = 1, 2, \cdots, K_1 \), where \( G_n = [g_{n1}, g_{n2}, \cdots, g_{nN_n}] \) and \( N_n \) is the number of antennas at \( D_n \). Since the channel coefficients are assumed to be generic, \( G_n \) has full rank and is invertible with probability 1. Therefore, \( \bar{U}_n \) is uniquely determined by \( U_n \), and vice versa. Then, solving (4) is equivalent to solving the following polynomial system
\[
\sum_{l=1}^{K_2} \bar{U}_l G_n^{-1} g_{ml} w_l f_{lm} V_m = 0, \ \forall m \neq n.
\]
\[
\hat{H}_{nm} = \sum_{l=1}^{K_2} \hat{U}_n^T G_n^{-1} g_{nl} \vec{w}_l f_{lm}^T \hat{V}_m = \left[ \begin{array}{c}
\hat{u}_{n,1}^T \\
\vdots \\
\hat{u}_{n,d_n}^T 
\end{array} \right] \left[ g_{n1}, \ g_{n2}, \ \cdots \ g_{nN} \right]^{-1} g_{n1} \vec{w}_1 f_{l1}^T \left[ \begin{array}{cccc}
\hat{v}_{m,1} & \hat{v}_{m,2} & \cdots & \hat{v}_{m,d_m} 
\end{array} \right] + \sum_{l=2}^{K_2} \left[ \begin{array}{c}
\hat{u}_{n,1}^T \\
\vdots \\
\hat{u}_{n,d_n}^T 
\end{array} \right] \left[ g_{n1}, \ g_{n2}, \ \cdots \ g_{nN} \right]^{-1} g_{n1} \vec{w}_1 f_{lm}^T \left[ \begin{array}{cccc}
\hat{v}_{m,1} & \hat{v}_{m,2} & \cdots & \hat{v}_{m,d_m} 
\end{array} \right]
\]

(38)

Assume a group of \(\{V_m, \hat{U}_n, \vec{w}\}\) is a feasible solution of (33). Then, \(\{\hat{V}_m, \hat{U}_n, \vec{w}\}\) is also a feasible solution of (33), where

\[
\vec{w} \triangleq \vec{w}/w_1 = \left[ 1, \ \frac{w_2}{w_1}, \ \cdots, \ \frac{w_{K_2}}{w_1} \right]^T,
\]

(34)

\[
\hat{V}_m \triangleq V_m P_m = \left[ \begin{array}{cccc}
\hat{v}_{m,1} & \hat{v}_{m,2} & \cdots & \hat{v}_{m,d_m}
\end{array} \right], \forall m,
\]

(35)

\[
\hat{U}_n \triangleq U_n Q_n = \left[ \begin{array}{cccc}
\hat{u}_{n,1} & \hat{u}_{n,2} & \cdots & \hat{u}_{n,d_n}
\end{array} \right], \forall n,
\]

(36)

and \(P_m\) is the inverse matrix of the first \(d_m\) rows of \(V_m\), \(Q_n\) is the inverse matrix of the first \(d_n\) rows of \(\hat{U}_n\). Therefore, solving (4) is equivalent to solving the following polynomial system

\[
\sum_{l=1}^{K_2} \hat{U}_n^T G_n^{-1} g_{nl} \vec{w}_l f_{lm}^T \hat{V}_m = 0, \forall m \neq n
\]

(37)

w.r.t. the variables \(\vec{w}_l (l \in \{2, \cdots, K_2\})\), \(\hat{v}_{m,p}\) \((p \in \{1, \cdots, d_m\})\), and \(\hat{u}_{n,q}\) \((q \in \{1, \cdots, d_n\})\).

The number of variables in (37) is given by \(N_{eq} = \sum_{m=1}^{K_1} d_m (M_m + N_m - 2d_m) + (K_2 - 1)\), and the number of equations is denoted by \(N_{eq} = \sum_{m=1}^{K_1} \sum_{m=1,m \neq n} d_m d_n\).

Denote the LHS of (37) as \(\hat{H}_{nm}\) for any \(m \neq n\), where \(\hat{H}_{nm}\) has a dimension of \(d_n \times d_m\), and the element in the \(i_m\)th column and \(j_n\)th row of \(\hat{H}_{nm}\) is denoted as \(H_{nm}^{i_mj_n}\) \((i_m \in \{1, \cdots, d_m\}, j_n \in \{1, \cdots, d_n\})\). Therefore, \(H_{nm}^{i_mj_n}\) is a polynomial w.r.t. the variables \(\vec{w}_l (l \in \{2, \cdots, K_2\})\), \(\hat{v}_{m,p}\) \((p \in \{1, \cdots, d_m\})\), and \(\hat{u}_{n,q}\) \((q \in \{1, \cdots, d_n\})\). All later mentioned polynomials are assumed to be w.r.t. the variables \(\vec{w}_l, \hat{v}_{m,p}\), and \(\hat{u}_{n,q}\). The expression of \(\hat{H}_{nm}\) is given by (38) on top of this page, where \(\vec{w}_l (l \in \{2, \cdots, K_2\})\), \(\hat{v}_{m,i_m}\) \((i_m \in \{1, \cdots, d_m\})\) and \(\hat{u}_{n,j_n}\) \((j_n \in \{1, \cdots, d_n\})\) are variables. Each element of \(\hat{H}_{nm} (l)\) in the third line of (38) at least contains one variable of \(\vec{w}_l (l \in \{2, \cdots, K_2\})\). Therefore, the constant terms of each element of \(\hat{H}_{nm}\) (i.e., \(H_{nm}^{i_mj_n}\)) are only determined by \(\hat{H}_{nm} (1)\) in the second line of (38). Since \(\left[ g_{n1}, \ g_{n2}, \ \cdots \ g_{nN} \right]^{-1} g_{n1} = [1, 0, \cdots, 0]^T\) and \(\vec{w}_1 = 1\), we have

\[
\hat{H}_{nm} (1) = \left[ \begin{array}{cccc}
f_{lm}^T & 0^T & \cdots & 0^T \\
0^T & \ddots & \cdots & 0^T \\
f_{1m} + f_{1m}^T \hat{v}_{m,1} & \cdots & f_{1m} + f_{1m}^T \hat{v}_{m,d_m} & 0^T \\
0^T & \ddots & \cdots & 0^T \\
\vdots & \ddots & \ddots & \vdots \\
0^T & \cdots & 0^T & 0^T 
\end{array} \right]
\]

(39)

where \(f_{lm}^T\) represents the \(i_m\)th element of the channel vector \(f_{lm}\), and \(f_{1m}^T = \left[ f_{1m}^{d_m+1}, f_{1m}^{d_m+2}, \cdots, f_{1m}^{M_m} \right]\). Therefore, the constant term of \(\hat{H}_{nm}^{i_mj_n}\) is zero for \(j_n > 1\), and is \(f_{1m}^{i_mj_n}\) for \(j_n = 1\).

We construct the following \(N_{eq}\) polynomials from (38),

\[
\mathcal{F}_{nm}^{i_mj_n} \triangleq \left\{ \begin{array}{ll}
\frac{f_{lm}^T - H_{nm}^{i_mj_n} \hat{v}_{m,1}}{f_{1m}^{i_mj_n}}, & m = 1, n = 2, j_n = 1, \\
\frac{H_{nm}^{i_mj_n} - H_{nm}^{i_mj_n}}{f_{1m}^T \hat{v}_{m,1}}, & m \geq 2, n = 1, j_n = 1, \\
\frac{H_{nm}^{i_mj_n} - H_{nm}^{i_mj_n}}{f_{1m}^T \hat{v}_{m,1}}, & m \geq 2, n \neq 1, j_n = 1, \\
\frac{H_{nm}^{i_mj_n}}{f_{1m}^T \hat{v}_{m,1}}, & j_n \geq 2,
\end{array} \right. 
\]

(40)

for \(\forall m \neq n\). The purpose to subtract some specific coefficients in (38) to form \(\mathcal{F}_{nm}^{i_mj_n}\) is to eliminate the constant terms of \(\hat{H}_{nm}^{i_mj_n}\), such that the constant terms of \(\mathcal{F}_{nm}^{i_mj_n}\) are all zero.

Since the linear operation on the polynomial system in (37) does not affect its solvability, the following polynomial system

\[
\mathcal{F}_{nm}^{i_mj_n} = \left\{ \begin{array}{ll}
f_{lm}, & m = 1, n = 2, j_n = 1, \\
0, & m \geq 2, n = 1, j_n = 1, \\
otherwise,
\end{array} \right. 
\]

(41)

is equivalent to the polynomial system in (37), i.e.,

\[
\hat{H}_{nm}^{i_mj_n} = 0, \forall m \neq n, i_m \in \{1, \cdots, d_m\}, j_n \in \{1, \cdots, d_n\}.
\]

(42)
It should also be noted that after the transformations, the coefficients of $\mathcal{F}_{j^m_{n^m}}$ are independent with $f_{1^m_{n^m}} (i_m \in \{1, \ldots, d_m\})$. This independency leads to the independency between $\mathcal{P} (\mathcal{F})$ in (48) and $f_{1^m_{n^m}}$, and finally leads to the contradiction in (49).

Next, we will prove that when $N_{\text{eq}} < N_{\text{eq}}$, (41) has no solution with probability 1.

When $d_m = M_m = N_m$ for all $m = 1, \ldots, K_1$, (41) is a group of linear equations. Similar to the single antenna case in [22], it is straightforward to show that when $N_{\text{eq}} < N_{\text{eq}}$, (41) has no solution with probability 1. Otherwise, (40) is a group of polynomials with $\deg (\mathcal{F}_{j^m_{n^m}}) \leq 2$, and at least $K_1 - 1$ of them have $\deg (\mathcal{F}_{j^m_{n^m}}) = 2$, where $\deg (\mathcal{F})$ denotes the degree of a polynomial $\mathcal{F}$. When $N_{\text{eq}} < N_{\text{eq}}$, we choose $N_{\text{eq}} + 1$ polynomials from (40) as a set $\mathcal{F}$, which contains the polynomials in (40) with $m = 1, n = 2, j_n = 1$ and $m \geq 2, n = 1, j_n = 1$. Then, the polynomials in $\mathcal{F}$ are algebraically dependent [26], which means there exists a nonzero polynomial $\mathcal{P}$ such that $\mathcal{P}(\mathcal{F}) = 0$.

In order to find a nonzero polynomial $\mathcal{P}$, we introduce the following results of [26].

Let $F_1, \ldots, F_{n^m+1} \in \mathbb{K}[X]$ be a sequence of $n + 1$ non-constant polynomials in $n$ variables $X = (X_1, \ldots, X_n)$ and let $\deg (F_i) = d_i$ for $i = 1, \ldots, n + 1$. Then, there exists a nonzero polynomial $\mathcal{P} = \mathcal{P}(Y) \in \mathbb{K}[Y]$ in $n$ variables $Y = (Y_1, \ldots, Y_{n+1})$ such that $\mathcal{P}(F_1, \ldots, F_{n^m+1}) = 0$, and if weight $(Y_i) = d_i$ for $i = 1, \ldots, n$ then weight $(\mathcal{P}) \leq d_1 \cdots d_{n^m+1}$. A polynomial $\mathcal{P}$ satisfying the above constraints is called Perron's relation between $F_1, \ldots, F_{n^m+1}$. Perron's relation $\mathcal{P}$ between $F_1, \ldots, F_{n^m+1}$ can be written in the form

$$\mathcal{P}(Y_1, \ldots, Y_{n^m+1}) = \sum_{(a_1, \ldots, a_{n^m+1}) \in \Delta} c_{a_1, \ldots, a_{n^m+1}} Y_1^{a_1} \cdots Y_{n^m+1}^{a_{n^m+1}},$$

where

$$\Delta = \{(a_1, \ldots, a_{n^m+1}) \in \mathbb{N}^{n+1}: d_1 a_1 + \cdots + d_{n+1} a_{n^m+1} \leq d_1 \cdots d_{n^m+1}\}.$$  
(43)

The coefficients $c_{a_1, \ldots, a_{n^m+1}}$ are obtained by solving a system of linear homogeneous equations. Specifically, let

$$C = (c_{a_1, \ldots, a_{n^m+1}} : (a_1, \ldots, a_{n^m+1}) \in \Delta)$$

and

$$\Delta^* = \{(b_1, \ldots, b_n) \in \mathbb{N}^n : b_1 + \cdots + b_n \leq d_1 \cdots d_{n+1}\}.$$  
(46)

Then,

$$\sum_{(a_1, \ldots, a_{n^m+1}) \in \Delta} c_{a_1, \ldots, a_{n^m+1}} L_{b_1, \ldots, b_n} (C) X_1^{b_1} \cdots X_n^{b_n}.$$  
(47)

Thus, $c_{a_1, \ldots, a_{n^m+1}}$ can be obtained by solving the system of linear homogeneous equations $L_{b_1, \ldots, b_n} (C) = 0$, where $(b_1, \ldots, b_n) \in \Delta^*$.

Therefore, similarly as (43), we can construct a nonzero polynomial $\mathcal{P}$ w.r.t. $\mathcal{F}_{j^m_{n^m}} \in \mathcal{F}$ as follows,

$$\mathcal{P}(\mathcal{F}) = \sum_{(a_1, \ldots, a_{n^m+1}) \in \Delta} c_{a_1, \ldots, a_{n^m+1}} \prod_{j^m_{n^m} \in \mathcal{F}} (\mathcal{F}_{j^m_{n^m}})^{a_{j^m_{n^m}}} = 0,$$

where $\prod_{j^m_{n^m} \in \mathcal{F}} (\mathcal{F}_{j^m_{n^m}})^{a_{j^m_{n^m}}}$ is a monomial w.r.t. $\mathcal{F}_{j^m_{n^m}} \in \mathcal{F}$, which corresponds to the monomial of $Y_1, \ldots, Y_{n^m+1}$ in (43). $a_{j^m_{n^m}}$ denotes the power of $\mathcal{F}_{j^m_{n^m}}$. $c_k$ denotes the coefficient of each monomial term of $\mathcal{P}$ and is independent with the values of $\{V_m, U_n, W\}$, which can be obtained similarly as $c_{a_1, \ldots, a_{n^m+1}}$ in (47). $A = \{a_{j^m_{n^m}} \mid \prod_{j^m_{n^m} \in \mathcal{F}} (\mathcal{F}_{j^m_{n^m}})^{a_{j^m_{n^m}}} \deg (\mathcal{F}_{j^m_{n^m}}) \leq \Delta \}$ is equivalent to $\Delta$ in (44), where $\Delta = \prod_{j^m_{n^m} \in \mathcal{F}} \deg (\mathcal{F}_{j^m_{n^m}})$.

If the polynomial system in (4) has solutions, then (41) holds. By substituting (41) into (48), we obtain

$$\mathcal{P} = \sum_{n=1}^{K_1} \sum_{i^m_{n^m} \leq d_m} \mathcal{D} (f_{1^m_{n^m}}) + \sum_{m=2}^{K_1} \sum_{n=1}^{d_m} \mathcal{D} (f_{1^m_{n^m}}) + \Delta = 0,$$

where $\Delta$ denotes the rest of the terms. Notice that the polynomial $\mathcal{P}$ is independent of $f_{1^m_{n^m}} (1 \leq i^m_{n^m} \leq d_m)$ and $\Delta$ only contains lower order terms of $f_{1^m_{n^m}} (1 \leq i^m_{n^m} \leq d_m)$. Since the channel gains $f_{1^m_{n^m}}$ are independent random variables, the second equality of (49) cannot hold with probability 1 unless $\mathcal{P}$ is identically zero, which contradicts the previous nonzero assumption. This implies that the polynomial system in (4) has no solution with probability 1 when $N_{\text{eq}} < N_{\text{eq}}$.  

**References**


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