A fuzzy group decision making approach for bridge risk assessment

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Abstract

This paper proposes a fuzzy group decision making (FGDM) approach for bridge risk assessment. The FGDM approach allows decision makers (DMs) to evaluate bridge risk factors using linguistic terms such as Certain, Very High, High, Slightly High, Medium, Slightly Low, Low, Very Low or None rather than precise numerical values, allows them to express their opinions independently, and also provides two alternative algorithms to aggregate the assessments of multiple bridge risk factors, one of which offers a rapid assessment and the other one leads to an exact assessment. A case study is investigated using the FGDM approach to illustrate its applications in bridge risk assessment. It is shown that the FGDM approach offers a flexible, practical and effective way of modelling bridge risks.

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1. Introduction

Bridge risk assessment is often conducted to determine the priority or the optimal scheme of bridge maintenance. For instance, Adey, Hajdin, and Brühwiler (2003) presented a risk-based approach to determining the optimal intervention for a bridge subject to multiple hazards. Johnson and Niezgoda (2004) presented a risk-based method for ranking, comparing and choosing the most appropriate bridge scour countermeasures using failure mode and effects analysis (FMEA) and risk priority numbers (PRNs). Stein, Young, Trent, and Pearson (1999) developed a risk-based method for assessing the risk associated with scour threat to bridge foundations. The risk of scour failure was defined as the product of the probability of scour failure or heavy damage and the cost associated with the failure, adjusted by a risk adjustment factor based on foundation and span types. Shetty, Chubb, Knowles, and Halden (1996) proposed a risk-based framework for assess-

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ment and prioritization of bridges in need of remedial work, which involves risk evaluation, rankings of bridges in terms of risk, design of remedial action for each bridge, and optimal allocation of resources for remedial work on different bridges. Risk is quantified as the product of probability of failure and consequences of failure. Lounis (2004) presented a risk-based approach for bridge maintenance optimization that takes into account several and possibly conflicting criteria, with emphasis on the risk of failure as a governing criterion. The optimal maintenance strategy was defined as the solution that achieved the best compromise among the three selected relevant and conflicting criteria: minimization of risk of failure, minimization of maintenance costs, and minimization of traffic disruption. Compromise programming was used to determine the optimal ranking of maintenance strategies in terms of their effectiveness in risk reduction, cost minimization, and traffic control. A multi-criteria optimality index was proposed as a measure of the effectiveness of the optimal maintenance strategy in achieving a satisfactory trade-off between the relevant and competing maintenance criteria.

Bridge risk is usually assessed against multiple criteria (or called risk factors) and by a group of decision makers (DMs) and is therefore a typical group decision making problem. Due to the fact that bridge risk cannot be precisely measured and can only be assessed using DMs’ knowledge and subjective judgments, it can be well modelled using linguistic variables and fuzzy set theory. In fact, fuzzy logic has been widely used to assess various risks such as construction project risk (Carr & Tah, 2001; Cho, Choi, & Kim, 2002; Kuchta, 2001), software development risk (Chen, 2001; Lee, 1996a; Lee, 1996b; Lee, 1999; Lee, Lee, Lee, & Chen, 2003), software operational risk (Xu, Khoshgoftaar, & Allen, 2003), forest fire risk (Iliadis, 2005), e-commerce development risk (Ngai & Wat, 2005), environmental risk (Sadiq & Husain, 2005), investment risk (Serguieva & Hunter, 2004), tourist risk (Tsaur, Tzeng, & Wang, 1997), and on. It seems to us, however, no attempt has been made so far to model bridge risk using fuzzy set theory. This paper aims at developing a practical and effective fuzzy group decision making (FGDM) approach and providing two alternative algorithms for bridge risk assessment.

In comparison with the existing approaches for bridge risk assessment, the proposed FGDM approach treats bridge risk assessment as a fuzzy multiple criteria group decision making problem. It allows a group of bridge experts (DMs) to make their judgments independently and express their opinions (judgments) in fuzzy linguistic terms rather than in precise numerical values that prove to be difficult in practice. So, the proposed FGDM approach will be easier to use and more realistic.

The paper is organized as follows: In Section 2, we briefly introduce some basic concepts on fuzzy sets, including fuzzy numbers, fuzzy arithmetics, defuzzification and fuzzy weighted average, to pave the way for the FGDM approach. In Section 3, we develop the FGDM approach to bridge risk assessment and provide two alternative algorithms to aggregate risk ratings. Section 4 investigates a case study using the proposed FGDM approach and algorithms to illustrate their potential applications in bridge risk assessment. Conclusions are offered in Section 5.

2. Basic concepts on fuzzy sets

Fuzzy sets are generalizations of crisp sets and were first introduced by Zadeh (1965) as a way of representing imprecise or vagueness in real world. A fuzzy set is a collection of elements in a universe of information where the boundary of the set contained in the universe is ambiguous, vague and otherwise fuzzy. Each fuzzy set is specified by a membership function, which assigns to each element in the universe of discourse a value within the unit interval [0, 1]. The assigned value is called degree (or grade) of membership, which specifies the extent to which a given element belongs to the fuzzy set or is related to a concept. If the assigned value is 0, then the given element does not belong to the set. If the assigned value is 1, then the element totally belongs to the set. If the value lies within the interval (0, 1), then the element only partially belongs to the set. Therefore, any fuzzy set can be uniquely determined by its membership function.

Let $X$ be the universe of discourse. A fuzzy set $\tilde{A}$ of the universe of discourse $X$ is said to be convex if and only if for all $x_1$ and $x_2$ in $X$ there always exists:

$$\mu_{\tilde{A}}(\lambda x_1 + (1 - \lambda) x_2) \geq \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)),$$

where $\mu_{\tilde{A}}(x)$ is the membership function of the fuzzy set $\tilde{A}$ and $\lambda \in [0, 1]$. 


A fuzzy set \( \tilde{A} \) of the universe of discourse \( X \) is said to be normal if there exists a \( x_i \in X \) satisfying \( \mu_{\tilde{A}}(x_i) = 1 \). Fuzzy sets can also be represented by intervals, which are called \( \alpha \)-level sets or \( \alpha \)-cuts. The \( \alpha \)-level sets \( A_\alpha \) of a fuzzy set \( A \) are defined as

\[
A_\alpha = \{ x \in X | \mu_{\tilde{A}}(x) \geq \alpha \} = \{ \min \{ x \in X | \mu_{\tilde{A}}(x) \geq \alpha \}, \max \{ x \in X | \mu_{\tilde{A}}(x) \geq \alpha \} \}. \tag{2}
\]

According to Zadeh’s extension principle (Zadeh, 1965), the fuzzy set \( \tilde{A} \) can be expressed as

\[
\tilde{A} = \cup_\alpha A_\alpha, \quad 0 < \alpha \leq 1. \tag{3}
\]

Fuzzy numbers are special cases of fuzzy sets that are both convex and normal. A fuzzy number is a convex fuzzy set, characterized by a given interval of real numbers, each with a grade of membership between 0 and 1. Its membership function is piecewise continuous and satisfies the following conditions:

(a) \( \mu_{A}(x) = 0 \) for each \( x \not\in [a,d] \);
(b) \( \mu_{A}(x) \) is non-decreasing (monotonic increasing) on \([a,b]\) and non-increasing (monotonic decreasing) on \([c,d]\);
(c) \( \mu_{A}(x) = 1 \) for each \( x \in [b,c] \),

where \( a \leq b \leq c \leq d \) are real numbers in the real line \( R = (-\infty, +\infty) \).

The most commonly used fuzzy numbers are triangular and trapezoidal fuzzy numbers, whose membership functions are respectively defined as

\[
\mu_A(x) = \begin{cases} 
    (x-a)/(b-a), & a \leq x \leq b; \\
    (d-x)/(d-b), & b \leq x \leq d; \\
    0, & \text{otherwise.}
\end{cases} \tag{4}
\]

\[
\mu_A(x) = \begin{cases} 
    (x-a)/(b-a), & a \leq x \leq b; \\
    1, & b \leq x \leq c; \\
    (d-x)/(d-c), & c \leq x \leq d; \\
    0, & \text{otherwise.}
\end{cases} \tag{5}
\]

For brevity, triangular and trapezoidal fuzzy numbers are often denoted as \((a,b,d)\) and \((a,b,c,d)\). It is obvious that triangular fuzzy numbers are special cases of trapezoidal fuzzy numbers with \( b = c \).

Let \( \tilde{A} = (a_1,a_2,a_3) \) and \( \tilde{B} = (b_1,b_2,b_3) \) be two positive triangular fuzzy numbers. Then basic fuzzy arithmetic operations on these fuzzy numbers are defined as (Dubois & Prade, 1980; Kaufmann & Gupta, 1991)

Addition: \( \tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3) \);
Subtraction: \( \tilde{A} - \tilde{B} = (a_1 - b_3, a_2 - b_2, a_3 - b_1) \);
Multiplication: \( \tilde{A} \times \tilde{B} \approx (a_1b_1, a_2b_2, a_3b_3) \);
Division: \( \tilde{A} \div \tilde{B} \approx (\frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_3}) \).

An important concept related to the applications of fuzzy numbers is defuzzification, which converts a fuzzy number into a crisp value. Such a transformation is not unique because different methods are possible. The most commonly used defuzzification method is the centroid defuzzification method, which is also known as center of gravity or center of area defuzzification. The centroid defuzzification method can be expressed as follows (Yager, 1981):

\[
\bar{x}_0(\tilde{A}) = \frac{\int_a^d x \mu_{\tilde{A}}(x) \, dx}{\int_a^d \mu_{\tilde{A}}(x) \, dx}, \tag{6}
\]

where \( \bar{x}_0(\tilde{A}) \) is the defuzzified value. For trapezoidal fuzzy numbers \((a,b,c,d)\), the centroid-based defuzzified value turns out to be

\[
\bar{x}_0(\tilde{A}) = \frac{1}{3} \left[ a + b + c + d - \frac{dc - ab}{(d+c)-(a+b)} \right]. \tag{7}
\]

Especially when \( b = c \), the above formula becomes
\[
x_0(\mathbf{A}) = \frac{a + b + d}{3},
\]  
which is the defuzzification formula of triangular fuzzy numbers \((a, b, d)\) and will be used in this paper.

The fuzzy weighted average of fuzzy numbers is referred to as fuzzy weighted average (FWA), which is defined as

\[
y = \frac{\bar{w}_1 \bar{x}_1 + \bar{w}_2 \bar{x}_2 + \cdots + \bar{w}_n \bar{x}_n}{\bar{w}_1 + \cdots + \bar{w}_n},
\]

where \(\bar{x}_1, \ldots, \bar{x}_n\) are \(n\) fuzzy numbers to be weighted and \(\bar{w}_1, \ldots, \bar{w}_n\) are fuzzy weights. Fuzzy arithmetic operations are found not suitable for computing \(\bar{y}\) because the weight variables appear in both denominator and numerator simultaneously. Lots of research has been done on how to compute \(\bar{y}\). The most commonly used approach is to calculate \(\bar{y}\) using the extension principle. Let \(x_{ia} = [x_{ia}^L, x_{ia}^U]\), \(w_{ia} = [w_{ia}^L, w_{ia}^U]\) and \(y_{ia} = [y_{ia}^L, y_{ia}^U]\) be the \(x\)-level sets of \(\bar{x}_i\), \(\bar{w}_i\) and \(\bar{y}_i\), respectively. Then \(y_{ia} = [y_{ia}^L, y_{ia}^U]\) can be derived by the following pair of fractional programming models:

\[
y_{ia}^L = \min \frac{w_{i1}x_{i1}^L + w_{i2}x_{i2}^L + \cdots + w_{in}x_{in}^L}{w_1 + w_2 + \cdots + w_n}
\]

\[
s.t. w_{ia}^L \leq w_i \leq w_{ia}^U, \quad i = 1, \ldots, n.
\]

\[
y_{ia}^U = \max \frac{w_{i1}x_{i1}^U + w_{i2}x_{i2}^U + \cdots + w_{in}x_{in}^U}{w_1 + w_2 + \cdots + w_n}
\]

\[
s.t. w_{ia}^L \leq w_i \leq w_{ia}^U, \quad i = 1, \ldots, n.
\]

Let

\[
z = 1/(w_1 + w_2 + \cdots + w_n),
\]

\[
v_i = zw_i, \quad i = 1, \ldots, n.
\]

The above fractional programming models can be simplified as (Kao & Liu, 2001)

\[
y_{ia}^L = \min v_1x_{i1}^L + v_2x_{i2}^L + \cdots + v_nx_{in}^L
\]

\[
s.t. v_1 + v_2 + \cdots + v_n = 1, \quad z \cdot w_{ia}^L \leq v_i \leq z \cdot w_{ia}^U, \quad i = 1, \ldots, n,
\]

\[
z \geq 0.
\]

\[
y_{ia}^U = \max v_1x_{i1}^U + v_2x_{i2}^U + \cdots + v_nx_{in}^U
\]

\[
s.t. v_1 + v_2 + \cdots + v_n = 1, \quad z \cdot w_{ia}^L \leq v_i \leq z \cdot w_{ia}^U, \quad i = 1, \ldots, n,
\]

\[
z \geq 0.
\]

These are linear programming models and are easy to solve using MS Excel Solver or LINDO software package.

3. The FGDM approach for bridge risk assessment

According to the British Highways Agency (2004), bridge risk can be expressed as any event or hazard that could hinder the achievement of business goals or the delivery of stakeholder expectations and is defined as the product of the likelihood and consequences of an event occurring. That is

\[
\text{Risk} = \text{Likelihood} \times \text{Consequences}.
\]

Risks associated with bridge structures maintenance activities include deterioration, failure to meet the Agency’s obligations for freedom of movement on the network and failure of a component, element or structure.

To identify and assess bridge risks, maintenance needs have to be identified, based on which projects can be developed, each project addressing either singular or multiple structures maintenance needs. All maintenance needs can be classified as being either:
Essential – work required to maintain safety standards. Work is required to be carried out on structures or structural elements because they are considered to be unsafe or structurally inadequate, e.g., major concrete repairs or replacement of structural elements.

Preventative – maintenance work that is not essential now but is justified on economic grounds as it provides minimum whole life cost maintenance. By timely intervention, preventative work will reduce the risk of essential work arising prematurely in the future, e.g., painting of steelwork.

Upgrading – work resulting from changes in requirements of faults, e.g., parapet replacement, pier upgrading.

Associated with the identified maintenance needs are the risk events that could occur if nothing were done about them in the short term of 3–4 years. The risk events can be defined as protective coating failure, deterioration, failure to meet obligation for required operational capacity, concrete spalling, equipment failure, non-structural component failure (waterproofing and non-structural joints), structural component failure (structural joints, bearings and parapets), component failure, element failure, or structural failure (global collapse). Each of them can be broken down further into a risk event chain.

For each defined risk event, its likelihood and consequences need to be assessed on the basis of evidence and engineering judgment. This can usually be done by a team of bridge experts or called decision makers (DMs) in the terminology of decision analysis. It turns out to be not easy, if not impossible, to require the DMs to provide precise numerical judgments about the likelihood and consequences of each risk event. So, fuzzy linguistic terms are much easier to be accepted and adopted by the DMs. Table 1 shows the linguistic terms defined for the likelihood of bridge risk event in this paper, whose membership functions are shown in Fig. 1. In the case that a risk event is broken down into an event chain, the likelihood of the risk event is defined to be the product of the likelihood ratings within the event chain.

Note that how to define the membership functions (particularly the parameters) of the linguistic terms is also a very important issue. It is most desirable that the DMs achieve a consensus on these definitions. If they disagree with each other, then average values should be used for the definitions.

The consequences of a risk event depend on the level and type of traffic using a structure or route, the features surrounding the structure (i.e. what it crosses or supports), and the availability of alternative routes, and might include human injury, network disruption, disruption to spur water or gas mains or other major utility supply lines, disruption to other transport networks adjacent to the structure (e.g. rail), repair/replacement of a structure/component/element, and environment damage.

<table>
<thead>
<tr>
<th>Likelihood rating</th>
<th>Description</th>
<th>Fuzzy number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Certain</td>
<td>Certainty (either already happened or certain to happen)</td>
<td>C = (1.0,1.0,1.0)</td>
</tr>
<tr>
<td>Very high</td>
<td>Very highly likely</td>
<td>VH = (0.85,1.0,1.0)</td>
</tr>
<tr>
<td>High</td>
<td>Highly likely</td>
<td>H = (0.7,0.85,1.0)</td>
</tr>
<tr>
<td>Slightly high</td>
<td>Likely</td>
<td>SH = (0.5,0.7,0.85)</td>
</tr>
<tr>
<td>Medium</td>
<td>Possible, and likely</td>
<td>M = (0.3,0.5,0.7)</td>
</tr>
<tr>
<td>Slightly low</td>
<td>Possible, but slightly unlikely</td>
<td>SL = (0.15,0.3,0.5)</td>
</tr>
<tr>
<td>Low</td>
<td>Possible, but unlikely</td>
<td>L = (0.0,0.15,0.3)</td>
</tr>
<tr>
<td>Very low</td>
<td>Possible, but very unlikely</td>
<td>VL = (0,0,0.15)</td>
</tr>
</tbody>
</table>

Fig. 1. Membership functions of likelihood ratings.
Eq. (16), we have

All the consequences are assessed against the following four criteria (see Fig. 2):

- Safety – safety of the public.
- Functionality – effects on the level of service/availability of the network for use.
- Sustainability – sustainability of both expenditure and workload, where the aim is to reach a state of steady expenditure and workload, avoiding the built-up of a backlog of unavoidable, essential work by doing effective, targeted preventative maintenance, i.e. painting steelwork, silane of new concrete and preventative maintenance on all bridges in new/as new condition. Sustainability will also require the timely replacement of structures.
- Environment – effects on the environment, including the (aesthetic) appearance of the structures.

Consequences must be credible and reasonable and should not be based on an extreme event. Table 2 shows the linguistic terms defined for the consequences of bridge risk event, whose membership functions are presented in Fig. 3. In the case that a risk event has multiple consequences, the most likely one should be used.

Let \( \tilde{R}_{ij}^{(k)} \) be the risk ratings of a risk event of the \( k \)th bridge structure under evaluation with respect to the four criteria, provided by DM\( k \) (the \( k \)th decision maker), \( k = 1, \ldots, m \), where \( m \) is the number of DMs. By Eq. (16), we have

\[
\tilde{R}_{ij}^{(k)} = \tilde{L}_{ij}^{(k)} \times \tilde{C}_{ij}^{(k)}, \quad i = 1, \ldots, n; \ j = 1, \ldots, 4; \ k = 1, \ldots, m, \quad (17)
\]

where \( \tilde{L}_{ij}^{(k)} \) is the likelihood rating of the risk event provided by DM\( k \) and is the same for the four criteria and \( \tilde{C}_{ij}^{(k)} \) \( (i = 1, \ldots, 4) \) are the consequences ratings of the risk event. Due to the fact that both \( \tilde{L}_{ij}^{(k)} \) and \( \tilde{C}_{ij}^{(k)} \) are fuzzy numbers, the risk ratings \( \tilde{R}_{ij}^{(k)} \) are also fuzzy numbers.

To determine the overall risk of the risk event, the relative importance weights of the four criteria need to be specified. This can be done by directly assigning a crisp or fuzzy weight or linguistic term to each criterion. Fig. 4 shows the linguistic terms defined for the relative importance weights of the four criteria.

Let \( \tilde{w}_{ij}^{(k)} \) \( (j = 1, \ldots, 4) \) be the fuzzy weights assigned by DM\( k \) to the four criteria. Note that crisp weights can be seen as a special case of fuzzy weights. The overall risk (or called aggregative risk) of each risk event can be determined in different ways. In what follows, we provide two alternative algorithms.

- **Algorithm-1**

1. Average likelihood and consequences ratings as well as the weights of criteria according to fuzzy addition operation:

\[
\tilde{L}_{ij} = \frac{1}{m} \sum_{k=1}^{m} \tilde{L}_{ij}^{(k)} = \left( \frac{1}{m} \sum_{k=1}^{m} L_{ij}^{(k)} \right), \quad i = 1, \ldots, n, \quad (18)
\]

\[
\tilde{C}_{ij} = \frac{1}{m} \sum_{k=1}^{m} \tilde{C}_{ij}^{(k)} = \left( \frac{1}{m} \sum_{k=1}^{m} C_{ij}^{(k)} \right), \quad i = 1, \ldots, n; \ j = 1, \ldots, 4, \quad (19)
\]

\[
\tilde{w}_{j} = \frac{1}{m} \sum_{k=1}^{m} \tilde{w}_{j}^{(k)} = \left( \frac{1}{m} \sum_{k=1}^{m} W_{j}^{(k)} \right), \quad j = 1, \ldots, 4, \quad (20)
\]
where $\tilde{R}_{ij}^{(k)} = (LR_{ijL}^{(k)}, LR_{ijM}^{(k)}, LR_{ijU}^{(k)})$, $\tilde{C}_{ij}^{(k)} = (LR_{ijL}^{(k)}, LR_{ijM}^{(k)}, LR_{ijU}^{(k)})$ and $\tilde{w}_{j}^{(k)} = (w_{jL}^{(k)}, w_{jM}^{(k)}, w_{jU}^{(k)})$ are respectively the triangular fuzzy numbers on likelihood and consequences ratings as well as the relative importance weights of criteria provided by DM\textsubscript{k}.

(2) Calculate risk ratings by Eq. (17) and fuzzy multiplication operation:

$$\tilde{R}_{ij} = \tilde{L}_{i} \times \tilde{C}_{ij}^{(k)} = (LR_{ijL}, LR_{ijM}, LR_{ijU} \times CR_{ijL}, LR_{ijM} \times CR_{ijM}, LR_{ijU} \times CR_{ijU}), \quad i = 1, \ldots, n; j = 1, \ldots, 4.$$
(3) Defuzzify the risk ratings and the weights of criteria by Eq. (8):

$$\bar{R}_{ij} = \frac{1}{3}(R_{ijL} + R_{ijM} + R_{ijU}), \quad i = 1, \ldots, n; \quad j = 1, \ldots, 4,$$

$$\bar{w}_j = \frac{1}{3}(w_{jL} + w_{jM} + w_{jU}), \quad j = 1, \ldots, 4,$$

where $\bar{R}_{ij} = (R_{ijL}, R_{ijM}, R_{ijU})$ and $\bar{w}_j = (w_{jL}, w_{jM}, w_{jU})$ are the triangular fuzzy numbers on risk ratings and the relative importance weights of criteria, respectively.

(4) Generate overall risk score by weighting and averaging the risk ratings:

$$RS_i = \sum_{j=1}^{4} \bar{w}_j \bar{R}_{ij} / \sum_{j=1}^{4} \bar{w}_j, \quad i = 1, \ldots, n.$$  

(5) Rank and prioritize bridge structures according to their overall risk scores: big risk score means high risk and high priority.

- **Algorithm-2**

(1) Average likelihood and consequences ratings and criteria weights by Eqs. (18)–(20).

(2) Compute the $\alpha$-level sets of the above averaged likelihood and consequences ratings and the weights of criteria by Eq. (2):

$$[LR]_z = [(LR)_{ijL}^L, (LR)_{ijU}^U] = [LR_{ijL} + \alpha(LR_{ijM} - LR_{ijL}), LR_{ijU} + \alpha(LR_{ijU} - LR_{ijM})],$$

$$[CR]_z = [(CR)_{ijL}^L, (CR)_{ijU}^U] = [CR_{ijL} + \alpha(CR_{ijM} - CR_{ijL}), CR_{ijU} + \alpha(CR_{ijU} - CR_{ijM})],$$

$$[w]_z = [(w)_L, (w)_U] = [w_{jL} + \alpha(w_{jM} - w_{jL}), w_{jU} + \alpha(w_{jU} - w_{jM})].$$

(3) Compute the $\alpha$-level set, $(RS) = [(RS)_z, (RS)_z]$, of the overall risk score by FWA:

$$[RS]_z^L = \text{Min} \frac{\sum_{j=1}^{4} w_j ((LR)_{ijL}^L \times (CR)_{ijL}^L)}{\sum_{j=1}^{4} w_j},$$

$$\text{s.t. } (w)_L \leq w_j \leq (w)_U, \quad j = 1, \ldots, 4,$$

$$[RS]_z^U = \text{Max} \frac{\sum_{j=1}^{4} w_j ((LR)_{ijL}^U \times (CR)_{ijU}^U)}{\sum_{j=1}^{4} w_j},$$

$$\text{s.t. } (w)_L \leq w_j \leq (w)_U, \quad j = 1, \ldots, 4,$$

which can be transformed into the following pair of linear programming (LP) models:

$$[RS]_z^L = \text{Min} \sum_{j=1}^{4} v_j ((LR)_{ijL}^L \times (CR)_{ijL}^L),$$

$$\text{s.t. } \sum_{j=1}^{4} v_j = 1,$$

$$v_j \cdot z \leq (w)_L, \quad (w)_L \cdot z \leq v_j \cdot z, \quad j = 1, \ldots, 4,$$

$$z \geq 0.$$

$$[RS]_z^U = \text{Max} \sum_{j=1}^{4} v_j ((LR)_{ijL}^U \times (CR)_{ijU}^U),$$

$$\text{s.t. } \sum_{j=1}^{4} v_j = 1,$$

$$v_j \cdot z \leq (w)_U, \quad (w)_U \cdot z \leq v_j \cdot z, \quad j = 1, \ldots, 4,$$

$$z \geq 0.$$
(4) Compute ranking index by the following equation (Chen & Klein, 1997):

\[ RI_i = \frac{\sum_{l=0}^{n_1} ([RS_i]^U_{x_l} - c] - \sum_{l=0}^{n_1} ([RS_i]^L_{x_l} - d]}{\sum_{l=0}^{n_1} ([RS_i]^U_{x_l} - (n+1)c - \sum_{l=0}^{n_1} ([RS_i]^L_{x_l} + (n-1)(d-c)), \quad i = 1, \ldots, n,} \]  

where \( c = \min_{i,l} \{ [RS_i]^L_{x_l} \}, \quad d = \max_{i,l} \{ [RS_i]^U_{x_l} \} \) and \( n_1 \) is the number of \( x \) levels minus one, satisfying \( 0 < x_1 < \cdots < x_{n_1} = 1 \).

(5) Rank and prioritize bridge structures according to their ranking indices: the bigger the ranking index, the higher the risk and the priority.

The difference between Algorithm-1 and Algorithm-2 is that the former changes a fuzzy multiple criteria decision making (MCDM) problem into a crisp (non-fuzzy) one through defuzzification, which is the most common and simplest way of dealing with fuzzy MCDM problems and has been widely used in fuzzy multiple criteria decision analysis (Carr & Tah, 2001; Chen, 2001; Cho et al., 2002; Lee, 1996a, 1996b, 1999; Lee et al., 2003; Ngai & Wat, 2005; Sadiq & Husain, 2005), while the latter solves the fuzzy MCDM problem using fuzzy extension principle and only defuzzifies the final results for comparison and/or ranking purpose. So, the former produces a crisp overall risk score for each bridge structure, while the latter yields a fuzzy overall risk score, which is thought to be the exact solution to the fuzzy MCDM problem under discussion.

Algorithm-1 proves to be simple, effective and easy to use and offers a rapid assessment to fuzzy MCDM problems. Algorithm-2 is relatively complicated, but provides an exact assessment to fuzzy MCDM problems and more information about assessment results. Users can choose either of the two algorithms or both of them to solve their problems.

In Algorithms-1 and 2, we average each DM’s opinions (ratings and criteria weights) equally, which implies that the DMs are equally important. In practice, if the DMs are of different importance, then their opinions should be weighted. The weights can be either crisp or fuzzy numbers like those defined by Fig. 4.

4. A case study

The British Highways Agency has identified thousands of risk events associated with bridge structures in the past. From their database, we randomly select five risk events associated with five bridge structures as our case study and consider three bridge experts as a group/team of DMs for simplicity. The three experts are asked to assess the five risk events against safety \((X_1)\), functionality \((X_2)\), sustainability \((X_3)\) and environment \((X_4)\) criteria independently using the fuzzy linguistic terms defined in Tables 1 and 2.

The two algorithms both average the likelihood and consequences ratings given by the three experts so that a group decision making problem can be simplified as a decision making problem with only one DM. The averaged ratings are also shown in Table 3.

Algorithm-1 converts the fuzzy MCDM problem into a crisp MCDM, which is shown in Table 5, where the weights are normalized. From the overall risk scores of the five bridge structures, it is clear that bridge structure 1 has the highest risk and should be given top priority for maintenance, followed by bridge structures 2 and 3. The priority ranking of the five bridge structures is obtained as \( BS_1 > BS_2 > BS_3 > BS_4 > BS_5 \).

To generate an exact fuzzy assessment for each bridge structure, eleven \( x \) levels are set for computation, i.e. 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0. Accordingly, eleven \( x \)-level sets are derived for the overall risk score of each bridge structure, which are shown in Table 6 and depicted in Fig. 5. The overall risk scores of the five bridge structures are all fuzzy numbers. This makes sense because the original ratings are all fuzzy numbers. Obviously, Algorithm-2 provides more information on the overall risk score of each bridge structure.
than Algorithm-1 despite the fact that it is more complicated in computation than the latter. The final priority ranking produced by Algorithm-2 is BS₁ > BS₂ > BS₃ > BS₄ > BS₅, which is the same as the priority ranking produced by Algorithm-1.

5. Conclusions

In this paper, we have developed a fuzzy group decision making approach called FGDM for bridge risk assessment. Bridge risk is defined as the product of the likelihood and consequences of a risk event that could occur if nothing were done about the bridge structure needing maintenance in the short term of 3–4 years. The consequences are assessed against safety, functional, sustainability and environment criteria, respectively. The FGDM approach allows a group of DMs to participate in the risk assessment, allows them to make
judgments independently, and also allows them to express their judgments on likelihood, consequences and the weights of criteria in linguistic terms rather than in precise numerical values that prove to be not easy. Two alternative algorithms have been developed to aggregate the assessments of the four criteria to generate an overall risk assessment. Algorithm-1 changes a fuzzy multiple criteria decision making problem into a crisp one and turns out to be simple enough and effective. Algorithm-2 handles fuzzy MCDM problems using the extension principle and provides exact solutions to the problems. Both algorithms are justified through a case study. It has been shown that the FGDM approach offers a flexible, practical and effective way of modelling bridge risks.

In some circumstances a bridge structure may have more than one risk event. In these situations, each risk event needs to be assessed separately and the maximum overall risk score should be assigned to the bridge structure. So, no matter how many risk events are involved in a bridge structure, the FGDM approach is always applicable.

Finally, we point out that one important aspect of group decision making is seeking to achieve a maximum consensus among the DMs. To this end, the proposed FGDM approach can be combined with the Delphi technique, which should be applied before implementing Algorithm-1 or 2. This will help to achieve a better and more credible risk priority ranking for bridge structures.

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References
