Adaptive position tracking control of permanent magnet synchronous motor based on RBF fast terminal sliding mode control

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1. Introduction

Recently, the permanent-magnet synchronous motors (PMSM) have been applied widely in the factory automation, household appliances, computer, high-speed aerospace drives, and automobiles because they have many advantages of high power density, high efficiency, high reliability and fast dynamics etc. However, the control performance of PMSM is still influenced by the variations of the mechanical parameters, the external load disturbances and perturbations in practical applications. It is difficult to accomplish the high-performance control of PMSM by using the conventional PID-type control methods.

Sliding mode control (SMC) is one of the effective control methods since it is insensitive to parameter variations, external disturbance injection and fast dynamic response [1–4]. It is successfully applied in many engineering systems, such as the motor drivers [1–9], networks [10–12], robot manipulators [13] and aircrafts [14] etc. In the theoretical research, the concept of SMC has been utilized to solve the uncertain system [15,16] and stochastic system [17] and has developed successfully in recent years.

To improve the performance of the motor system, many kinds of SMC have been presented and attempted to apply in the motor drivers. In general, sliding mode manifolds are designed to be the linear hyper plane which is called linear sliding mode (LSM) in the literatures [1–4]. The control system using the linear sliding mode cannot guarantee the convergence in finite time. In the literatures [5–9], some integral operations are utilized as the sliding mode manifolds to accomplish the motor control, and the insensitivity of the systems to uncertainties and disturbances are obtained as well. However, the systems conversation lags behind the terminal sliding mode (TSM) control [18–20]. The states of TSM control system which are nonlinear can guarantee converging to the origin in finite time. Hence, the terminal sliding mode control is applied in some motor driver system [18–20] to improve the performance of the system. But the TSM control has some disadvantages. Firstly, TSM cannot provide good convergence performance if the system state is far away from the equilibrium. Secondly, the convergence time of the TSM control is longer than that of the LSM control if the system state is far away from the equilibrium. And thirdly, the chattering phenomenon cannot be avoided because of its discontinuity item. Therefore, the fast terminal sliding mode (FTSM) controller attracts many concerns from many researchers, which retains the merits of the LSM and the TSM while reducing the chattering phenomenon [21]. However, the performance of FTSM controller in Ref. [21] depends on some parameters of the systems, which decreases the robustness of the system. Especially, it is difficult to attain the parameters of the system as PMSM in the real applications. So its applications in the real PMSM control system are restricted and affected.

The neural network is effective to solve the problems related to unknown nonlinear systems. In some literatures [22–24], the
tracking control by using the neural network for PMSM has become an attracting topic. However, the network control in the motor system has the approximation errors in the state region. Hence, in this paper, a neural adaptive sliding mode control algorithm is proposed to accomplish the position tracking of the field-oriented control for PMSM. The proposed algorithm is designed by combining FTSM with the radial basis function (RBF), which not only compensates the network approximation errors in the state region but also solves the problem that FTMS depends on the parameters of the PMSM. As a result, the algorithm improves the performance of the PMSM control system, such as the tracking accuracy, robustness and response time, etc. The parameters of neural network are updated according to the Lyapunov approach which is used to prove the stability of the closed-loop system. The computer simulation results are presented to verify the feasibility and effectiveness of the method.

2. Dynamic mode of PMSM with deadzone

2.1. PMSM model

In the rotating (d–q) reference frame, the mathematics model of permanent-magnet synchronous motor is shown as below:

\[
\begin{align*}
\dot{i}_d &= -\frac{L}{2} i_q + p_n i_i q + \frac{u_d}{L} \\
\dot{i}_q &= -\frac{L}{2} i_d - \frac{R_s}{2} i_q - \frac{L}{2} i_d \omega + \frac{u_q}{L} \\
\dot{\omega} &= \frac{2}{L} i_d x_1 - \frac{2}{L} \omega - \frac{1}{L} \frac{2}{L} \\
\dot{\theta} &= \omega
\end{align*}
\]

where \(i_d, i_q\) and \(u_d, u_q\) are the currents and voltages (volt) of the motor d axis and q axis respectively; \(R_s\) is the stator resistance of the motor (ohm); \(L\) is the self inductance of the motor stator (H); \(\psi_f\) is the permanent magnet flux of the motor (volt s); \(T_i\) is the motor torque (N m); \(p_n\) is the pole-pairs of motor; \(B\) and \(J\) are the viscous friction coefficient and the inertia constant of motor; \(\omega\) and \(\theta\) are the angular velocity (rad/s) and the rotor position of motor (rad).

2.2. The PMSM position track control base on FOC

The system is designed for the three closed-loop control system to accomplish the position of motor by the field-oriented control technology, and then the high performance is obtained. The three closed-loop control system includes the position control loop, the speed control loop and the current control loop. Moreover, the method of the \(i_d=0\) vector control is utilized in order to simplify the motor control system. In the condition of \(i_d=0\), the Eq. (1) can be simplified as follows:

\[
\begin{align*}
\dot{i}_d &= -L p_n i_i q \\
\dot{i}_q &= -\frac{R_s}{2} i_q - \frac{L}{2} i_d \omega + \frac{u_q}{L} \\
\dot{\omega} &= \frac{2}{L} i_d x_1 - \frac{2}{L} \omega - \frac{1}{L} \frac{2}{L} \\
\dot{\theta} &= \omega
\end{align*}
\]

The control system is regarded as a linear system, and the overall configuration of the speed vector control of PMSM is shown in Fig. 1, which consists of the PMSM, the SVPWM voltage source inverter, the power source rectifier, the automatic current regulator (ACR) of the motor, the encoder used to detect speed and position, the speed controller and the position tracking controller. The position tracking controller will be described in detail in the next section.

3. FTSM controller designed

A position tracking control algorithm is proposed to accomplish the position tracking control of the PMSM, which is designed based on the fast terminal sliding mode control theory. The control item is continuous in the algorithm, so the chattering phenomenon can be avoided compared to some conventional sliding mode control methods.

For the convenience of designing the controller, assume that the speed control loop, the current control loop and the inverter are ideal. So the dynamics of the PMSM mathematics model is simplified as the following second-order differential Eq. (3).

\[
\begin{align*}
\dot{\theta} &= \omega \\
\dot{\omega} &= \frac{2}{L} i_d x_1 - \frac{2}{L} \omega - \frac{1}{L} \frac{2}{L}
\end{align*}
\]

The position controller of the PMSM is designed to attain the highly precise position tracking to the reference position and be robust to the load disturbance. Assume that the signal of the reference position is \(\theta^*\), and it is sufficiently smooth and has the second-order derivative almost everywhere. So define the error state equation as follows:

\[
e = \theta^* - \theta
\]
\[\dot{e} = \dot{\theta} - \theta\]

where \(\theta\) is the reference position of the motor, and \(\dot{\theta}\) is the actual position of the motor.

In order to improve the position tracking precision and attain the good dynamic performance, a fast terminal sliding mode surface is designed as follows [25]:

\[S = \dot{e} + \alpha e + \beta \frac{de}{dt}\]  

(6)

where \(\alpha, \beta > 0\) are constant, and \(p, q\) are odd numbers and \(p > q\).

**Theorem 1.** : the position error system (4) can converge to zero in finite time while (6) is chosen as the sliding mode manifold and the control law is designed as follows:

\[l_q = l_{eq} + l_{sw}\]

with

\[l_{eq} = \left(\frac{J}{p_B q f}\right) \left(\dot{\theta}^* + \frac{B}{J} \omega + \alpha e + \beta \frac{de}{dt}\right)\]

\[l_{sw} = \phi S + \gamma S^{q_0}/p_0\]

(7)

(8)

where \(p_0 > q_0\) and \(p_0, q_0\) are odd numbers, \(\phi, \gamma > 0\) are constant.

**Proof.** : when the Lyapunov function \(V < 0\), the sliding mode control exists and the error system can converge. So the Lyapunov function can be defined as:

\[V = \frac{1}{2} S^2\]

(10)

According to (6), the \(S\) is obtained as below:

\[\dot{S} = \dot{e} + \alpha e + \beta \frac{de}{dt} = \dot{\theta}^* - \omega + \alpha e + \beta \frac{de}{dt}\]

(11)

Then substituting (3) into (11), the \(S\) is written as:

\[\dot{S} = \ddot{\theta}^* - \frac{p_B q f}{J} l_q + \frac{B}{J} \omega + \frac{T_l}{J} + \alpha e + \beta \frac{de}{dt}\]

(12)

The following equation is obtained from the (7)-(9) and (12):

\[\dot{S} = \ddot{\theta}^* - \frac{p_B q f}{J} (l_q + l_{sw}) + \frac{B}{J} \omega + \frac{T_l}{J} + \alpha e + \beta \frac{de}{dt}\]

\[= \ddot{\theta}^* - \frac{p_B q f}{J} \left(\theta^* + \frac{B}{p_B q f} \alpha e + \beta \frac{de}{dt}\right) + \frac{B}{J} \omega + \frac{T_l}{J} + \alpha e + \beta \frac{de}{dt}\]

\[+ \phi S + \gamma S^{q_0}/p_0\]

\[= \ddot{\theta}^* - \frac{p_B q f}{J} \left(\theta^* - \frac{B}{p_B q f} \alpha e + \beta \frac{de}{dt}\right) + \phi S + \gamma S^{q_0}/p_0\]

\[+ \frac{B}{J} \omega + \frac{T_l}{J} + \alpha e + \beta \frac{de}{dt}\]

So

\[S = \frac{p_B q f}{J} (-\phi S - \gamma S^{q_0}/p_0) + \frac{T_l}{J}\]

(13)

As a result:

\[\dot{V} = SS = S\left(-\phi S - \gamma S^{q_0}/p_0 + \frac{T_l}{J}\right)\]

(14)

where \(\phi = \frac{p_B q f}{J} \alpha e + \beta \frac{de}{dt}\) and \(\gamma = \gamma_1 + \gamma_12\). \(\gamma_11 > 0, \gamma_12 > 0\). So

\[\dot{V} = SS\]

\[= S\left(-\phi S - \gamma_11 S^{q_0}/p_0 - \gamma_12 S^{q_0}/p_0 + \frac{T_l}{J}\right)\]

\[= S\left(-\phi S - \gamma_11 S^{q_0}/p_0 - \gamma_12 S^{q_0}/p_0 + \frac{T_l}{J}\right)\]

\[= -\phi_1 ||S||^2 - \gamma_11 ||S||^{q_0}/p_0 - ||S||^{q_0}/p_0 (\gamma_12 - \frac{T_l}{J} S^{q_0}/p_0)\]

Owing to the designed parameter is satisfied:

\[\gamma_12 > |T_l/j(S^{q_0}/p_0)|\]

(15)

The inequation can be obtained as follows:

\[\dot{V} < S(-\phi S - \gamma_12 S^{q_0}/p_0)\]

\[= -\phi_1 ||S||^2 - \gamma_11 ||S||^{q_0}/p_0 < 0\]

(16)

Moreover, the study in Ref. [25] proves that the states can reach the FTSM manifold \(S = 0\) within finite time. Once \(S\) reaches zero, it will stay at zero, and \(e\) will converge to zero within finite time. The time from \(e(0) \neq 0\) to \(e = 0\) is:

\[t_s = \frac{p}{2(p-q)} \ln \left(\frac{\alpha e(0)^{p-q}/p + \beta}{\beta}\right)\]

(17)

Therefore, the error state Eq. (3) converges to zero in finite time. This case completes the proof. The structure of the proposed control algorithm can be described in Fig. 2. □

**Remark 1.** : in applications \(p, q, \text{and} \beta\) can be chosen to regulate the convergence speed of \(S\) according to Eq. (17).

**Remark 2.** : the above algorithm is attained in the ideal states when the \(\psi_1, J\) and \(B\) are known. Actually, these parameters are difficult to be achieved in the real PMSM control system.

4. **RBF-FTSM controller designed**

In this section, a novel position control algorithm based on the RBF fast terminal sliding mode control is proposed to accomplish the position tracking control of PMSM owing to the above question. The algorithm combines the fast terminal sliding mode control with RBF so that it has not only the advantages of the fast terminal sliding mode control but also the good performance of the neural network control which relies on the math model of the plant.

The total structure is shown in Fig. 3, which has the same structure as (6). The equivalent controller is attained by the RBF network.

The sliding mode manifold is still defined as (6) in the control system. In order to design the neural-network-based controller, the adjustment of the weights and the analysis of the error convergence are proposed in the following **Theorem 2**.

(Fig. 2. The structure of position tracking control based on FTSMC.)
Theorem 2: : the position error system (4) can converge to zero in finite time, while (6) is chosen as the sliding mode manifold, and the control law is designed as follows:

\[ i_q = \dot{i}_eq + i_{sw} \]  

with

\[ \dot{i}_eq = W^T H = \sum_{j=1}^{m} w_j h_j(x) \]  

\[ i_{sw} = \phi S + \gamma S^{\phi_0/\mu_0} \]  

where \( x_i = [S] \); \( \phi > 0, \gamma > 0 \) is the gain of the switch item; \( h_j(x) = \exp(-(|x_i - c_j|^2/b_j)^2) \) and the weights are adjusted by using the following control law:

\[ d\dot{W} = -\eta_1 S \left( \frac{p_n \psi_f}{2} \right) H(x) \]  

where the adaptive gains \( \eta_1 > 0, \eta_2 > 0 \) and \( b_j \in R \) are predetermined in (8).

Proof: : when the Lyapunov function \( V < 0 \), the sliding mode control exists and the error system can converge. So the Lyapunov function can be defined as according to the literatures [6,24]:

\[ V = \frac{1}{2} S^T S + \frac{1}{2} \eta_1^{-1} W^T \dot{W} \]  

where \( \dot{W} = W^* - \dot{W}, \ddot{W} = -\dot{W} \).

Take the time derivative of (22), and obtain:

\[ V = S^T \dot{S} + \eta_1^{-1} W^T \ddot{W} \]  

According to (6), \( \dot{S} \) is rewritten as below:

\[ \dot{S} = \ddot{e} + \beta (p/q) e^{\phi_1 - 1} \dot{e} \]  

From (24) and (3), (4), \( \dot{S} \) is obtained as follows:

\[ \dot{S} = \left( \dot{\theta}^* - \frac{p_n \psi_f}{2} i_{eq} + B \frac{\omega}{2} + T_1 \right) + \dot{e} \dot{e} + \beta (p/q) e^{\phi_1 - 1} \dot{e} \]  

And by substituting (18) into (25), the equation is obtained in (26):

\[ \dot{S} = \left( \dot{\theta}^* - \frac{p_n \psi_f}{2} i_{eq} + B \frac{\omega}{2} + T_1 \right) + \dot{e} \dot{e} + \beta (p/q) e^{\phi_1 - 1} \dot{e} \]  

From the definition of the equivalent controller, the equation is shown as:

\[ \ddot{\theta}^* - \frac{p_n \psi_f}{2} i_{eq} + B \frac{\omega}{2} + \dot{e} \dot{e} + \beta (p/q) e^{\phi_1 - 1} \dot{e} = 0 \]  

\( \dot{S} \) is obtained as (28) according to Eq. (26) and Eq. (27).

\[ \dot{S} = \frac{p_n \psi_f}{2} \left( i_{eq} - \dot{i}_{eq} - i_{sw} + T_1 \right) \]  

There exits coefficients \( w^* \) such that \( \dot{i}_{eq} \) approximates \( i_{eq} \) that is [26]:

\[ \max_{x \in X} \| i_{eq}(x,W^*) - i_{eq} \| \leq \mu \quad \mu > 0 \]

Let \( i_{eq} = W^* H(x) + \mu \), hence:

\[ \dot{V} = \frac{S}{\eta_1} \left( \frac{p_n \psi_f}{2} \right) \left( i_{eq} - \dot{i}_{eq} - \phi S - \gamma S^{\phi_0/\mu_0} + T_1 \right) \]

\[ = \frac{1}{\eta_1} \frac{p_n \psi_f}{2} \left( W^T H(x) + \mu_2 - \dot{W}^T H - \phi S - \gamma S^{\phi_0/\mu_0} + T_1 \right) \]

\[ = \frac{1}{\eta_1} \frac{p_n \psi_f}{2} \left( W^T H(x) + \mu_2 - \phi S - \gamma S^{\phi_0/\mu_0} + T_1 \right) \]

\[ S = \frac{S}{\eta_1} \left( \frac{p_n \psi_f}{2} \right) \left( \mu_2 + T_1 - \phi S - \gamma S^{\phi_0/\mu_0} \right) \]

Suppose \( \phi_1 = \frac{p_n \psi_f}{2} \phi, \gamma_1 = \frac{p_n \psi_f}{2} \gamma \) and \( \gamma_1 = \gamma_{11} + \gamma_{12} \) and the parameter \( \gamma_{11} \) satisfies \( \gamma_{11} \geq \left( \frac{\mu + \mu_2}{\mu_0} \right) \) in Theorem 2, the following inequation can be obtained:

\[ V < \frac{S}{\eta_1} \left( \phi S - \gamma S^{\phi_0/\mu_0} \right) \]

\[ = - \phi_1 \| S \| - \gamma_{12} \| S \| \left( \frac{\phi_0}{\mu_0} \right) < 0 \]

The remark that \( e \) converges to zero within finite time is the same as the Theorem 1. \( \square \)

5. Simulation and experiment

In this section, the computer simulations are conducted in order to validate the feasibility and effectiveness of the proposed method. The computer simulation is mainly utilized to verify the performance of the RBF-FTSM controller of the PMSM position tracking compared to PD and FTSMS controller. The simulation environment is the Matlab/Simulink. The Simulink model of the PMSM vector control system, which includes the proposed RBF-FTSM, FTSMS and PD controller, has been constructed. The parameters of the PMSM are shown as in Table 1.

The simulation experiment includes three parts, i.e. the position tracking with external disturbances, the position tracking...
when $J$ has a perturbation and the PMSM start response. The parameters of the RBF-FTSM controller are designed as follows: $\alpha = 2, \beta = 2, \phi = 100, \gamma = 200, q = 5, p = 9$ and $q_0 = 1, p_0 = 3$. And the parameters of the FTSM controller are same as those of RBF-FTSM.

5.1. Position tracking with external disturbances

The experiments are conducted in order to evaluate the tracking performance of the RBF-FTSM controller and sensitiveness to the

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Motor parameters in the simulation.</th>
</tr>
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<tbody>
<tr>
<td>Power ($P$)</td>
<td>1.0 [KW]</td>
</tr>
<tr>
<td>Torque ($T$)</td>
<td>1 [N m]</td>
</tr>
<tr>
<td>Speed ($w$)</td>
<td>1500 [r/min]</td>
</tr>
<tr>
<td>Stator Resistor ($R_s$)</td>
<td>2.875 [ohm]</td>
</tr>
<tr>
<td>Stator inductor ($L_q, L_d$)</td>
<td>0.0085 [H]</td>
</tr>
<tr>
<td>Rotor inertia ($J$)</td>
<td>0.001 [Kg m$^2$]</td>
</tr>
<tr>
<td>Poles ($P$)</td>
<td>4</td>
</tr>
<tr>
<td>Flux ($\phi_f$)</td>
<td>0.175 [Wb]</td>
</tr>
</tbody>
</table>

**Fig. 4.** The result of the motor position tracking: (a) motor position tracking, (b) motor position tracking error, and (c) phase plane of $e$ and $\dot{e}$. 
The experiments are tested in the presence of the sinusoid external disturbances which are $\Delta d = 1.6 \sin(t)$ and the given position tracking curve $n^* = \sin(6\pi t)$.

Fig. 4 shows the results of the motor position tracking in the condition of RBF-FTSM, FTSM and PD control. As shown in Fig. 4, the RBF-FTSM control provides better tracking performance than FTSM and PD. Its error of the position tracking is smallest among three methods and the convergence time is shorter than FTSM.

5.2. Position tracking with system parameters perturbation

The experiment is used to testify the robustness of the presented method. The results of the simulations are shown in Fig. 5.

Fig. 5 shows the experimental results of the PMSM position tracking control when $J$ has a perturbation, i.e. $\Delta J = 0.5J$ and the external disturbances are the same as the above test. In the test, when the system uncertainty quantity is too large, the FTSM

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**Fig. 5.** The result of the motor position tracking when $J$ has a perturbation: (a) motor position tracking, (b) motor position tracking error, and (c) phase plane of $e$ and $\dot{e}$. 
control cannot converge in the finite time. But RBF-FTSM control can still converge in the same condition. Hence, the performance of the RBF-FTSM is better than FTSM and PD. The test testifies that the RBF-FTSM has good robustness.

5.3. PMSM control system response

The response of the present control methods is evaluated in the experiment. Fig. 6 shows the simulation experimental results of PMSM start response by using three control algorithms. From the results, the system response by using the RBF-FTSM is faster than that by using other control algorithms.

As the above three tests, the RBF-FTMS control has better performance, such as better robustness, start response, convergence and position tracking and so on.

6. Conclusion

This paper has proposed a neural adaptive sliding mode control algorithm to accomplish the position tracking control in

Fig. 6. The experimental results of PMSM start response: (a) motor start response, (b) position tracking error at motor starting, and (c) phase plane of $e$ and $\dot{e}$. 
the PMSM vector control system. The proposed algorithm is designed by combining FTSM with the radial basis function (RBF), which not only compensates the network approximation errors in the state region but also solves the problem of FTSM controller design whose performance depends on the parameters of the PMSM. Hence, it can be effective when the parameters of the PMSM are unknown and further improve the performance of the PMSM control system. The neural network parameters are updated according to the Lyapunov approach which is employed to guarantee the stability of the closed-loop system. The simulation experiment results indicate that the proposed algorithm can improve the performance of the PMSM control system such as fast response, high robustness and precise tracking position, etc. In future works, the authors intend to apply the proposed algorithm in other areas, such as the uncertain system as published in the literatures [10–12,15–17].

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